

Spectral Density of (Pseudo)Scalar Currents at Finite Temperature

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Abstract

We study the spectral densities of (pseudo)scalar currents at finite temperature in general case when mass of two quarks are different. Such spectral densities are necessary for the phenomenological investigation of hadronic parameters. We use quark propagator at finite temperature and show that an additional branch cut arises in spectral density, which corresponds to particle absorption from the medium. The obtained results at $T \rightarrow 0$ limit are in good agreement with the vacuum results.

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1 Introduction

Heavy ion collision experiments provide an opportunity to investigate particle properties in the medium. Inspired by these experiments, there is an increasing interest to investigate the properties of hadronic matter under extreme condition [1],[2]. In general, the media created by these collisions consist different mesons and baryons. Investigation of hadronic properties at finite temperature and density directly using the fundamental thermal QCD Lagrangian is highly desirable. However, such interactions occur in a region very far from the perturbative regime, where the quark-gluon coupling constant becomes large and the perturbative methods is not suitable for calculation of these properties. Therefore, we need nonperturbative approaches.

Some nonperturbative approaches are Lattice QCD, Heavy Quark Effective Theory (HQET), QCD Sum Rules, etc. Among these approaches, the QCD Sum Rules method [3] and its extension to finite temperature [4] has been widely used as an efficient and applicable tool to

investigate the hadronic properties [5]. The main elements in QCD sum rules approach are correlation function and dispersion relation. The correlation function is an object with dual nature. At large negative q^2 , it can be evaluated by perturbative methods, whereas at positive q^2 , it must be represented in terms of hadronic observables. Dispersion relation allows us to link the thermal correlator for positive values of q^2 to the one for negative values. Spectral functions in different cases were studied in the literature [6]-[13].

In the present work, we investigate the two-point thermal correlator using the real time formulation of the thermal field theory [14]. We calculate the spectral densities of (pseudo)scalar currents at finite temperature, which are necessary for the phenomenological investigation of (pseudo)scalar mesons parameters. We use quark propagator at finite temperature and show how an additional branch cut corresponding to particle absorption from the medium arises in spectral density. We also compare our results with predictions obtained at zero temperature.

2 Thermal Spectral Densities of (Pseudo) Scalar Currents

We begin by considering the thermal correlation function,

$$\Pi(q, T) = i \int d^4x e^{iq \cdot x} \langle \mathcal{T}(J(x)J^+(0)) \rangle, \quad (1)$$

where $J(x) =: \bar{q}_1(x)\Gamma q_2(x)$ is the interpolating current that carries the quantum numbers of the state concerned and \mathcal{T} indicates the time ordered product. Here $\Gamma = I$ or $i\gamma_5$ for scalar and pseudoscalar particles, respectively. The thermal average of the operator, $A = \mathcal{T}(J(x)J^+(0))$, appearing in the above thermal correlator is expressed as

$$\langle A \rangle = \text{Tr} e^{-\beta H} A / \text{Tr} e^{-\beta H}, \quad (2)$$

where H is the QCD Hamiltonian, and $\beta = 1/T$ stands for the inverse of the temperature. Traces are carried out over any complete set of states. In the real time version, thermal correlator has the form of a 2×2 matrix. However, this matrix may be diagonalized, when it is expressed by a single analytic function, which determines completely the dynamics of the corresponding two-point function [15]. As this function is simply related to any one, say the 11-component of the matrix, we need to calculate only this component of the correlation function. The 11-component of the thermal quark propagator is a sum of vacuum quark propagator expression and a term depending on the Fermi distribution function,

$$S(q) = (\gamma^\mu q_\mu + m) \left(\frac{1}{q^2 - m^2 + i\varepsilon} + 2\pi i n(|q_0|) \delta(q^2 - m^2) \right), \quad (3)$$

where $n(x)$ is the Fermi distribution function, $n(x) = [\exp(\beta x) + 1]^{-1}$. Now, we proceed to obtain the temperature-dependent dispersion relation. The time ordering product in Eq. (1) can be expressed as

$$\langle T(J(x)J^+(x')) \rangle = \theta(x_0 - x'_0) \langle J(x)J^+(x') \rangle + \theta(x'_0 - x_0) \langle J^+(x')J(x) \rangle, \quad (4)$$

where $\theta(x)$ is step function.

Using Kubo-Martin-Schwinger relation, $\langle J(x_0)J^+(x'_0) \rangle = \langle J^+(x'_0)J(x_0 + i\beta) \rangle$ for thermal expectation and making Fourier and some other transformations, we get the following expression for the thermal correlation function in momentum space [14]:

$$\Pi(|\mathbf{q}|, q_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dq'_0 M(|\mathbf{q}|, q'_0) \left(\frac{1}{q_0 - q'_0 + i\varepsilon} - \frac{\exp(-\beta q_0)}{q_0 - q'_0 - i\varepsilon} \right), \quad (5)$$

where

$$M(|\mathbf{q}|, q_0) = \int d^4x e^{iq \cdot x} \langle J(x)J^+(0) \rangle. \quad (6)$$

In the above transformations, the following standard integral representation for the θ -step function is used:

$$\theta(x_0 - x'_0) = \frac{1}{2i\pi} \int_{-\infty}^{\infty} dk_0 \frac{\exp[ik_0(x_0 - x'_0)]}{k_0 - i\varepsilon}. \quad (7)$$

The imaginary part of the correlation function can be simply evaluated using the formula $\frac{i}{x+i\varepsilon} = \pi\delta(x) + iP(\frac{1}{x})$, which leads to [16]:

$$\Pi(q, T) = \int_0^{\infty} ds \frac{\rho(s)}{s + Q_0^2}, \quad (8)$$

where $\rho(q, T) = \frac{1}{\pi} \text{Im}\Pi(q, T) \tanh \frac{\beta q_0}{2}$ and $Q_0^2 = -q_0^2$. In some cases, the correlation function has ultraviolet divergent. If the spectral density does not vanish at $s \rightarrow \infty$, the dispersion integral in Eq. (8) diverges. A standard way to overcome this problem is to subtract first few terms of its Taylor expansion at $q^2 = 0$ from $\Pi(q, T)$. The thermal correlation function in momentum space can be written as

$$\Pi(q, T) = -i \int \frac{d^4k}{(2\pi)^4} \text{Tr}(\Gamma S(k) \Gamma S(k - q)), \quad (9)$$

where $\Gamma = I$ and $i\gamma_5$ for scalar and pseudoscalar particles, respectively. Inserting propagators from Eq. (3) in Eq. (9) and carrying out the k_0 integration, we obtain the imaginary part of $\Pi(q, T)$ in the following form:

$$\begin{aligned} \text{Im}\Pi(q, T) = & -N_c \int \frac{d\mathbf{k}}{8\pi^2} \frac{1}{\omega_1 \omega_2} \left[(\omega_1^2 - \mathbf{k}^2 + \mathbf{k} \cdot \mathbf{q} - \omega_1 q_0 \pm m_1 m_2) \right. \\ & \times [(1 - n_1 - n_2 + 2n_1 n_2) \delta(q_0 - \omega_1 - \omega_2) - (n_1 + n_2 - 2n_1 n_2) \delta(q_0 - \omega_1 + \omega_2)] \\ & + (\omega_1^2 - \mathbf{k}^2 + \mathbf{k} \cdot \mathbf{q} + \omega_1 q_0 \pm m_1 m_2) \\ & \left. \times [(1 - n_1 - n_2 + 2n_1 n_2) \delta(q_0 + \omega_1 + \omega_2) - (n_1 + n_2 - 2n_1 n_2) \delta(q_0 + \omega_1 - \omega_2)] \right], \quad (10) \end{aligned}$$

where m_1 and m_2 are quark masses, $\omega_1 = \sqrt{\mathbf{k}^2 + m_1^2}$, $\omega_2 = \sqrt{(\mathbf{k}-\mathbf{q})^2 + m_2^2}$, $n_1 = n(\omega_1)$, $n_2 = n(\omega_2)$ and the plus and minus signs in front of m_1 , m_2 , correspond to scalar and pseudoscalar particles, respectively. The term, which does not include the Fermi distribution functions, show the vacuum contribution. Terms including the Fermi distributions depict medium contributions. The delta-functions in the different terms of Eq. (10) control the regions of non-vanishing imaginary parts of $\Pi(q, T)$, which define the position of the branch cuts. As seen the term including $\delta(q_0 - \omega_1 - \omega_2)$ gives contribution when $q_0 = \omega_1 + \omega_2$. Using Cauchy-Schwarz inequality, $(\sum_{i=1}^n a_i^2)(\sum_{i=1}^n b_i^2) \geq (\sum_{i=1}^n a_i b_i)^2$ we see that,

$$\omega_1 \omega_2 = \sqrt{\mathbf{k}^2 + m_1^2} \sqrt{(\mathbf{k}-\mathbf{q})^2 + m_2^2} \geq |\mathbf{k}| |\mathbf{k}-\mathbf{q}| + m_1 m_2, \quad (11)$$

and for $q_0 = \omega_1 + \omega_2$, we get,

$$q_0^2 = m_1^2 + \mathbf{k}^2 + m_2^2 + (\mathbf{k}-\mathbf{q})^2 + 2\omega_1 \omega_2 \geq (m_1 + m_2)^2 + \mathbf{q}^2. \quad (12)$$

Therefore, we obtain the first branch cut, $q^2 \geq (m_1 + m_2)^2$, which coincides with zero temperature cut describing the standard threshold for particle decays. This term survives at zero temperature and it is called the annihilation term. On the other hand, the term including $\delta(q_0 - \omega_1 + \omega_2)$ gives contribution when $q_0 = \omega_1 - \omega_2$. Similarly to the above expression, we obtain,

$$q_0^2 = m_1^2 + \mathbf{k}^2 + m_2^2 + (\mathbf{k}-\mathbf{q})^2 - 2\omega_1 \omega_2 \leq (m_1 - m_2)^2 + \mathbf{q}^2, \quad (13)$$

and therefore an additional branch cut arises at finite temperature, $q^2 \leq (m_1 - m_2)^2$, which corresponds to particle absorption from the medium. It is called scattering term and vanishes at $T = 0$.

In the following, we restrict our calculations with $|\mathbf{q}| = 0$, when there is no angular dependence. Note that, with $|\mathbf{q}| = 0$, the value of $|\mathbf{k}|$, fixed by the δ -functions in Eq. (10), is the magnitude of three momentum of quark or antiquark in the center-of-mass of the quark-antiquark system:

$$\mathbf{k}^2 = \frac{(q_0^2 - (m_1 + m_2)^2)(q_0^2 - (m_1 - m_2)^2)}{4q_0^2}. \quad (14)$$

In the $|\mathbf{q}| = 0$ case, as it is seen, the terms including $\delta(q_0 - \omega_1 - \omega_2)$ and $\delta(q_0 + \omega_1 + \omega_2)$ functions in Eq. (10), give contributions at the regions, $q_0 \geq (m_1 + m_2)$ and $q_0 \leq -(m_1 + m_2)$, respectively giving the vacuum cuts. Similarly, the terms including $\delta(q_0 - \omega_1 + \omega_2)$ and $\delta(q_0 + \omega_1 - \omega_2)$ functions in Eq. (10), give contributions at in the regions, $0 \leq q_0 \leq (m_1 - m_2)$ and $-(m_1 - m_2) \leq q_0 \leq 0$, respectively giving the Landau cuts. After straightforward calculations, we find the vacuum

part of the $Im\Pi(q, T)$ as:

$$Im\Pi(q_0, T = 0) = \frac{N_c}{8\pi q_0^2} \sqrt{(q_0^2 - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2} (q_0^2 - (m_1 - m_2)^2). \quad (15)$$

Taking into account both branch cuts after some transformations, the annihilation and scattering parts of spectral density is found as:

$$\rho_{a,pert}(s, T) = \rho_0(s) \left[1 - n\left(\frac{\sqrt{s}}{2} \left(1 + \frac{m_1^2 - m_2^2}{s}\right)\right) - n\left(\frac{\sqrt{s}}{2} \left(1 - \frac{m_1^2 - m_2^2}{s}\right)\right) \right], \quad (16)$$

for $(m_1 + m_2)^2 \leq s \leq \infty$,

$$\rho_{s,pert}(s, T) = \rho_0(s) \left[n\left(\frac{\sqrt{s}}{2} \left(1 + \frac{m_1^2 - m_2^2}{s}\right)\right) - n\left(-\frac{\sqrt{s}}{2} \left(1 - \frac{m_1^2 - m_2^2}{s}\right)\right) \right], \quad (17)$$

for $0 \leq s \leq (m_1 - m_2)^2$, with $m_1 \geq m_2$. Here, $\rho_0(s)$ is the spectral density in the lowest order of perturbation theory at zero temperature and it is given by

$$\rho_0(s) = \frac{3}{8\pi^2 s} q^2(s) v^n(s), \quad (18)$$

where $q(s) = s - (m_1 - m_2)^2$ and $v(s) = \left(1 - 4m_1 m_2 / q(s)\right)^{1/2}$. Here $n = 3$ and $n = 1$ for scalar and pseudoscalar particles, respectively. As it is seen, at $T \rightarrow 0$ limit these expressions are in good consistency with the vacuum expressions. Moreover, the obtained results are well consistent with the existing results in $m_1 = m_2$ case [17]-[19] for the scalar and pseudoscalar particles.

As an example, we present the dependence of the annihilation and scattering parts of the spectral density for K^\pm and D^\pm particles in Figs. 1 and 2. In numerical analysis, we use the values $m_s = 0, 13$ GeV and $m_c = 1, 46$ GeV for the quark masses. As it is clear, in the region of the standard threshold for particle decays, the $\rho_0(s)$ is replaced by the annihilation term. In the case of light mesons, the values of $\rho_{a,pert}(s, T)$ considerably differ from those of the $\rho_0(s)$. However, in the case of heavy mesons, the $\rho_{a,pert}(s, T)$ and $\rho_0(s)$ values are very close to each other. From Fig. 1, we also see that the in light K^\pm cases, the medium contributions play important role and consist higher percentage of the total value.

Our concluding result is that the thermal contributions contribute significantly to the spectral function.

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References

- [1] K. Yagi, T. Hatsuda and Y. Miake, Quark-Gluon Plasma, Cambridge University (2005).
- [2] J. Letessier, J. Rafelski, Hadrons and Quark-Gluon Plasma, Cambridge University (2002).
- [3] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147, 385 (1979). M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147, 448 (1979).
- [4] A. I. Bochkarev and M. E. Shaposhnikov, Nucl. Phys. B268, 220 (1986).
- [5] P. Colangelo, A. Khodjamirian, In: At the Frontier of Particle Physics, vol.3, ed. M. Shifman, World Scientific, Singapore, 1495 (2001).
- [6] R. Rapp and J. Wambach, Adv. Nucl. Phys. 25, 1 (2000).
- [7] H. Leutwyler and A. Smilga, Nucl. Phys. B342, 302 (1990).
- [8] H. van Hees and R. Rapp, Phys. Lett. B 606, 59 (2005).
- [9] S. Sarkar, B. K. Patra, V. J. Menon and S. Mallik, Indian J.Phys. 76A 385-391 (2002).
- [10] S. Mallik, S. Sarkar, Eur.Phys. J.C 61:489-494 (2009).
- [11] S. Ghosh, S. Sarkar and S. Mallik, arXiv:hep-ph/1004.2162v2.
- [12] E. V. Veliev, G. Kaya, Acta Phys. Pol. B 41, 1905 (2010).
- [13] E. V. Veliev, K. Azizi, H. Sundu, N. Aksit, arXiv:hep-ph/1010.3110.
- [14] A. Das, Finite Temperature Field Theory, World Scientific (1999).
- [15] R.L. Kobes and G.W. Semenoff, Nucl. Phys. 260, 714 (1985).
- [16] S. Mallik and K. Mukherjee, Phys. Rev. D58, 096011 (1998).
- [17] C.A. Dominguez, M. Loewe, J.C. Rojas, JHEP 08, 040 (2007).
- [18] C. A. Dominguez, M. Loewe, J. C. Rojas, Y. Zhang, arXiv:hep-ph/1010.4172.
- [19] E. V. Veliev, H. Sundu, K. Azizi, M. Bayar Phys. Rev. D 82, 056012 (2010).

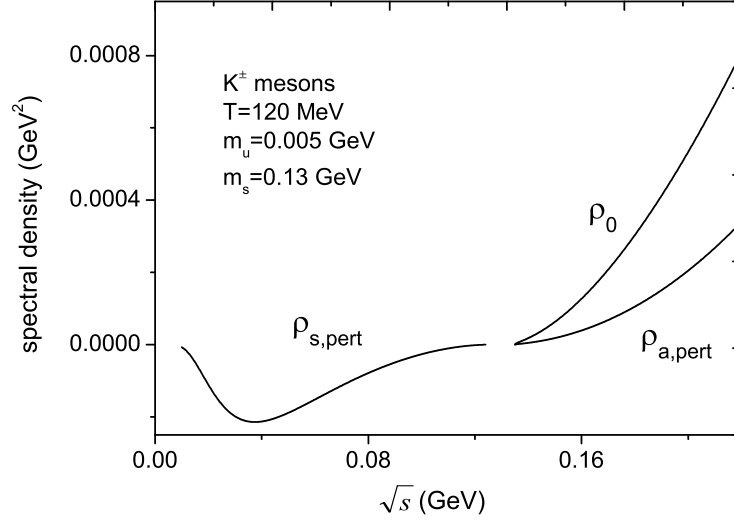


Figure 1: The dependence of the spectral density of K^\pm meson at temperature $T = 120$ MeV on the \sqrt{s} parameter.

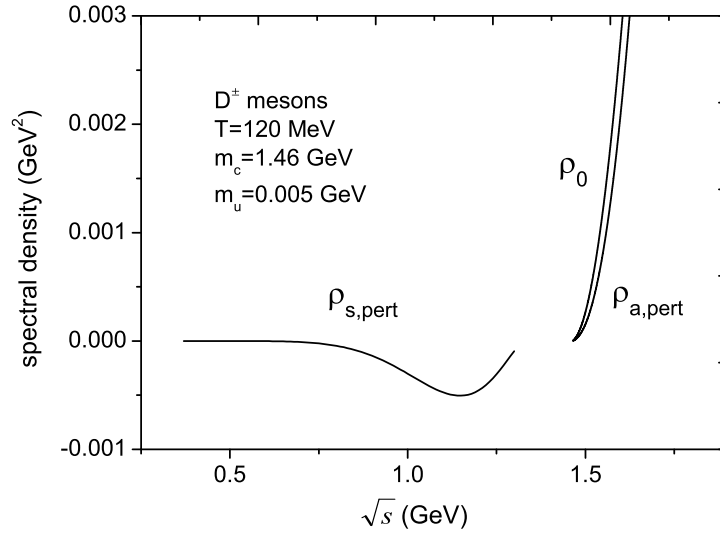


Figure 2: The dependence of the spectral density of D^\pm meson at temperature $T = 120$ MeV on the \sqrt{s} parameter.